

Time Allowed: 3 hours

Q1. (a) Solve the following linear system by Gauss's elimination method.

$$y + z = -2, 6y + 6z = -12, x + y + z = 2.$$

(b) Solve the following linear system by Gauss-Jordan elimination method.

$$x + 2y + 3z = 5, 2x + 5y + 3z = 3, 4x + 4z = 17.$$

Q2. (a) Solve the following linear system of equations by Cramer's rule.

$$4x + 3y = 12, 2x + 5y = -8.$$

(b) Find invers of the matrix $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$

Q3. (a) Find the rank of matrix, $A = \begin{bmatrix} 0 & 4 & 1 \\ 2 & 6 & -2 \\ 4 & 8 & -5 \end{bmatrix}$

(b) Find the Eigen values of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

Q4 (a) Show that the following matrices A and B are symmetric

$$A = \begin{bmatrix} 1 & -3 & 4 \\ -3 & 2 & -5 \\ 4 & -5 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 & 6 & 7 \\ 6 & -8 & 3 \\ 7 & 3 & 1 \end{bmatrix}$$

(b) For the vectors $a = (4, 0, 1)$ and $b = (2, -5, 1)$ find $2a - 2b$.

Q5. (a) Using dot product, find the angle between vectors $u = (1, 2, 3)$ and $v = (4, -5, 6)$. Do the vectors form

an acute angle, right angle, or obtuse angle?

(b) Calculate the cross product for the vectors $a = (1, 1, 0)$ and $b = (3, 0, 0)$.

Q6. (a) Find the real and imaginary part of complex number $(\sqrt{2} + i)^2$

(b) Find the Divergence of the vector function $v = [3xz, 2xy, -yz^2]$

Q7. (a) Find the Curl of the vector field $v = [yz, 3zx, z]$

(b) Find the general solution of differential equation $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 6y = 0$

Q8. (a) Find the Laplace transform of the following function:

$$f(t) = t^2 - 2t$$

(b) Given $F(S) = L(f(t))$, find $f(t)$

$$F(S) = \frac{2S + 16}{S^2 - 16}$$

Paper : **Methods of Applied Mathematics**

Examination:

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gadgets are not allowed.

Q1. (a) Solve the following linear system by Gauss's elimination method.

$$2x - 7y + 4z = 9,$$

$$x + 9y - 6z = 1,$$

$$-3x + 8y + 5z = 6.$$

(b) Solve the following linear system by Gauss Jordan elimination method.

$$2x + 2y + 4z = 18,$$

$$x + 3y + 2z = 13,$$

$$3x + y + 3z = 14.$$

Q2. (a) Solve the following linear system of equations by Crammer's rule.

$$x + 2z = 6$$

$$-3x + 4y + 6z = 30$$

$$-x - 2y + 3z = 8.$$

(b) Find invers of the matrix $A = \begin{bmatrix} 0 & 4 & 1 \\ 2 & 6 & -2 \\ 4 & 8 & -5 \end{bmatrix}$

Q3. (a) Find the rank of matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 4 \\ 6 & 4 & 8 \end{bmatrix}$

(b) Find the Eigen values of the matrix $A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

NOTE: Attempt any five questions from the rest. All questions carry equal marks. Phones and other Electronic Gadgets are not allowed.

Q1. Solve the following system of linear equations by

(a) Gaussian elimination method (b) Gauss - Jordan elimination method

$$\begin{aligned}x - 3y + z &= 4, \\2x - 8y + 8z &= -2, \\-6x + 3y - 15z &= 9.\end{aligned}$$

Q2. (a) Use Cramer's Rule to solve the following system of linear equations

$$\begin{aligned}x + y &= 5 \\2x + 3y &= 8\end{aligned}$$

(b) Find determinant of the matrix $A = \begin{bmatrix} 2 & 0 & 3 \\ -1 & 4 & -2 \\ 1 & -3 & 5 \end{bmatrix}$.

Q3. (a) Find the modulus and argument of the complex number $z = 3 + 4i$

(b) Find the Eigen values of the matrix $A = \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix}$

Q4. (a) Let $A = \begin{bmatrix} -5 & 2 \\ -7 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 2 & 7 \end{bmatrix}$. Determine whether $AB = BA$

(b) Prove that the matrix $A = \begin{bmatrix} 0 & -5 & 4 \\ 5 & 0 & -1 \\ -4 & 1 & 0 \end{bmatrix}$ is skew symmetric.

Q5. (a) For the vectors $u = (2, -1, 1)$, $v = (-4, 0, 5)$ and $w = (0, 3, -8)$. Find $3u + v - 4w$.

(b) Calculate the dot product of vectors $u = (1, 2, 3)$ and $v = (4, -5, 6)$. Do the vectors form an acute angle, right angle, or obtuse angle?

Q6. (a) Determine whether the vectors $v_1 = (2, 2, 0)$, $v_2 = (1, -1, 1)$ and $v_3 = (4, 2, -2)$ are linear dependent or independent?

(b) Find the Gradient of the scalar function $f = x^4 + y^4$

Q7. (a) Find the Divergence of the vector function $V = [\sin xy, \sin xy, z \cos xy]$

(b) Find the Curl of the vector field $V = [y^n, z^n, x^n]$, ($n > 0$, integer)

Q8. (a) Find the Laplace transform of the following function:

$$f(t) = t^2 - 2t$$

(b) Given $F(s) = L(f(t))$, find $f(t)$ $F(s) = \frac{4s - 3\pi}{s^2 + \pi^2}$

Q4. (a) Determine whether the vectors $v_1 = (2, 2, 0)$, $v_2 = (1, -1, 1)$ and $v_3 = (4, 2, -2)$ are linear dependent or independent?

(b) Let $u = (2, -1, 1)$, $v = (-4, 0, 5)$ and $w = (0, 3, -8)$. Find $u + v - 4w$.

Q5. (a) Calculate the dot product of vectors $u = (1, 2, 3)$ and $v = (4, -5, 6)$. Do the vectors form an acute angle, right angle, or obtuse angle?.

(b) Calculate the cross product between $a = (3, -3, 1)$ and $b = (4, 9, 2)$.

Q6. (a) Find the Gradient of the scalar function $f = x^2 + y^2$

(b) Find the Divergence of the vector function $[x^3 + y^3, 3xy^2, 3zy^2]$

Q7. (a) Find the Curl of the vector field $[y, 2x^2, 0]$

(b) Find the general solution of differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 0$

Q8. (a) Find the Laplace transform of the following function:

$$f(t) = a + bt + ct^2$$

(b) Given $F(S) = L(f(t))$, find $f(t)$

$$F(S) = \frac{4S - 3\pi}{S^2 + \pi^2}$$