Differential Equations Past Papers 1

NOTE: *Attempt **any five questions**. All questions carry equal marks. Phones and other Electronic Gadgets are*

Paper : **Differential Equations** Examination:

Time Allowed : 3 hours Total Marks: 70, Passing Marks (35)

$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+5y=0$$

(b) Find the general solution of differential equation with variable coefficients

$$\frac{d^3y}{dx^3}-\frac{dy}{dx}=e^x$$

Q2: Determine whether $x=0$ is a regular singular point of the differential equation

$$2x^2y''+7x(x+1)y'-3y=0$$

Q3: Use the method of Frobenious method to find the one solution near $x=0$ of $x^2y''+xy'+x^2y=0$

Q4: (a) Find the general solution in terms of j_v *and* Y_{-v} of $x^2y''+xy'+(x^2-25)y=0$

(b) Find the general solution in terms of j_v *and* j_{-v} of $y''+\frac{1}{x}y'+\left(1-\frac{n^2}{x^2}\right)y=0$

Q5: Find the Legendre polynomials $P_2(x), P_3(x), P_4(x), P_5(x)$ *and* $P_6(x)$ by the application of recurrence Formula $nP_n(x)=(2n-1)xP_{n-1}(x)-(n-1)P_{n-2}(x)=0$ where $P_\circ(x)=1$ *and* $P_1(x)=x,\ n=2,3,4,5$

Q6: Find the Laguere polynomials $L_2(x), L_3(x), L_4(x), L_5(x)$ *and* $L_6(x)$ by the three-term recurrence Relation $L_n(x)=\frac{2n-1-x}{n}L_{n-1}(x)-\frac{n-1}{n}L_{n-2}(x)$ where $L_\circ(x)=1$ *and* $L_1(x)=1-x,\ n=2,3,4,5,6$

Q7: Find the Chebyshev polynomials defined $T_{n+1}(x)=2xT_n(x)-T_{n-1}(x)$ *for* $n\geq 1$. If $T_\circ(x)=1,\ T_1(x)=x$, then show that $T_2(x)=2x-1,\ T_3(x)=4x^3-3x$ *and* $T_4(x)=8x^4-8x^2+1$, n=1, 2, 3

Q8: (a) Find the general solution of the system of differential equations $\frac{dx}{dt}=3y$ *and* $\frac{dy}{dt}=2x$

(b) Solve the Sturm-Lioville problem $Y''+\lambda Y=0$ with conditions $Y(0)+Y'(0)=0,\ Y(\pi)+Y'(\pi)=0$

Differential Equations Past Papers 2

Q1: (a) Find the general solution of differential equation with constant coefficients

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}-y=0$$

(b) Find the general solution of differential equation with variable coefficient

$$\frac{d^3y}{dx^3}-\frac{dy}{dx}=e^x$$

Q2: Determine whether $x=0$ is a regular singular point of the differential equation

$$2x^2y''+7x(x+1)y'-3y=0$$

Q3: Use the method of Frobenious method to find the one solution near $x=0$ of $2x\,y''+(x+1)y'+3y=0$

Q4: (a) Find the general solution in terms of j_v *and* Y_{-v} of $x^2y''+xy'+\left(x^2-\frac{1}{16}\right)y=0$

(b) Find the general solution in terms of j_v *and* j_{-v} of $x^2y''+xy'+\left(9x^2-\frac{1}{9}\right)y=0$

Q5: Find the Hermite polynomials $H_2(x), H_3(x), H_4(x), H_5(x)$ *and* $H_6(x)$ by the three-term recurrence Relation $H_{n+1}(x)=2xH_n(x)+2H_{n-1}(x)$ where $H_\circ(x)=1$ *and* $H_1(x)=1-x,\ n=1,2,3,4,5$

Q6: Find the Laguere polynomials $L_2(x), L_3(x), L_4(x), L_5(x)$ *and* $L_6(x)$ by the three-term recurrence Relation $L_n(x)=\frac{2n-1-x}{n}L_{n-1}(x)-\frac{n-1}{n}L_{n-2}(x)$ where $L_\circ(x)=1$ *and* $L_1(x)=1-x,\ n=2,3,4,5,6$

Q7: (a) Find the general solution of the system of differential equations $\frac{dx}{dt}=3y$ *and* $\frac{dy}{dt}=2x$

(b) Solve the Sturm-Lioville problem $Y''+\lambda Y=0$ with conditions $Y(0)+Y'(0)=0,\ Y(\pi)+Y'(\pi)=0$

Q8: Find the Chebyshev polynomials of first and second kind defined by $T_n(x)=\cos(n\cos^{-1}x)$ and $U_n(x)=\frac{\sin[(n+1)\cos^{-1}x]}{\sqrt{1-x^2}}$ .Show that $T_\circ(x)=1,\ T_1(x)=x$ *and* $T_2(x)=2x^2-1$

$U_\circ(x)=1,\ U_1(x)=2x$ *and* $U_2(x)=4x^2-1,\quad n=0,1,2$

Differential Equations Past Papers 3

Q1. (a) Find the general solution of differential equation with constant coefficients

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}-y=0$$

(b) Find the general solution of differential equation with variable coefficients

$$\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=e^{3x}$$

Q2. Find the series solution near $x=0$ of the differential equation

$$2x^2y''+7x(x+1)y'-3y=0$$

Q3. Use the method of Frobenious method to find the one solution near $x=0$ of

$$9x^2y''+3x^2y'+2y=0$$

Q4. (a) Find the general solution in terms of j_v *and* Y_{-v} if $x^2y''+xy'+(x^2-25)y=0$

(b) Find the general solution in terms of j_v *and* j_{-v} if $y''+\frac{1}{x}y'+\left(1-\frac{n^2}{x^2}\right)y=0$

Q5. Find the Lengendre polynomials $P_2(x), P_3(x), P_4(x), P_5(x)$ *and* $P_6(x)$ by the application of recurrence formula $nP_n(x) = (2n-1)x P_{n-1}(x) - (n-1)2P_{n-2}(x)$ where $P_\circ(x) = 1$ and $P_1(x) = x$ For $n = ,2,3,4,5.$

Q6. Find the Hermit polynomials $H_2(x), H_3(x), H_4(x), H_5(x)$ *and* $H_6(x)$ by the three-term recurrence relation $H_{n+1}(x) = 2xH_n(x) + 2H_{n-1}(x)$ where $H_\circ(x) = 1$ and $H_1(x) = 1 - x$ For $n = 1,2,3,4,5.$

Q7. (a) Find the general solution of the system of differential equations

$$\frac{dx}{dt} = 3y - 2x \; and \; \frac{dy}{dt} = 2x - y$$

(b) Solve the Sturm-Lioville problem $Y'' + \lambda Y = 0$ with the conditions $Y(0) + Y'(0) = 0,\; Y(\pi) + Y'(\pi) = 0$

Q8. Find the Chebyshev polynomials of first and second kind defined by $T_n(x) = \cos(n\cos^{-1} x)$ and $U_n(x) = \dfrac{\sin[(n+1)\cos^{-1} x]}{\sqrt{1-x^2}}$. Show that

$T_\circ(x) = 1,\; T_1(x) = x,\; T_2(x) = 2x - 1$ and $U_\circ(x) = 1,\; U_1(x) = 2x,\; U_2(x) = 2x^2 - 1$ *for* $n = 0,1,2.$